Pre-Calculus Honors - Quarter 1, Test 2

Name: ___________________________ Date: _______________ Period: ___

Directions: The use of a calculator is NOT permitted on this exam.

For numbers 1-4, circle your answer to each question. No partial credit will be given.

1) If \( f(x) = 5x - \frac{2}{7} \), the value of \( \frac{f(b)-f(a)}{b-a} \) is

(A) 5  (B) 1  (C) 0  (D) -1  (E) -5

\[
\frac{5b - \frac{2}{7} - \left(5a - \frac{2}{7}\right)}{b-a} = \frac{5b-5a}{b-a} \cdot \frac{5}{b-a} \cdot \frac{5(b-a)}{b-a}
\]

2) Which inequality is equivalent to the statement "\( x \) is at most 3 units from 5"?

(A) \(|3-x| \geq 5\)  (B) \(|x-5| \geq 2\)  (C) \(|x-3| \leq 5\)

(D) \(|5-x| \leq 3\)  (E) \(|x+5| \leq 2\)

3) The line parallel to the line \(2x - 3y = 5\) that passes through the point \((-7,4)\) would have the equation:

(A) \(y - 4 = -\frac{2}{3}(x + 7)\)  (B) \(y - 4 = -\frac{2}{3}(x + 7)\)  (C) \(y - 4 = \frac{2}{3}(x + 7)\)

(D) \(y - 4 = -\frac{3}{2}(x + 7)\)  (E) None of the preceding

4) If \( h(x) = \sqrt{x^2 + 9} \), \( f(x) = \sqrt{x} \), and \( g(x) = x^2 + 9 \), which of the following is a correct composition?

a. \( h(x) = (g \circ f)(x) \)

b. \( f(x) = (g \circ h)(x) \)

c. \( h(x) = (f \circ g)(x) \)

d. \( g(x) = (f \circ h)(x) \)
Factor $a^6 + b^9$

$(a^2 + b^3)(a^4 - a^2b^3 + b^6)$
For numbers 5-14, show all work in the space provided. Partial credit will be given when appropriate. A correct answer with incorrect or no supporting work will not receive full credit.

5) Solve and express your answer on a number line AND in interval notation:

\[
\frac{x + 6}{x - 1} \leq 3
\]

\[
\frac{x + 6}{x - 1} - 3 \cdot \frac{x - 1}{x - 1} \leq 0
\]

\[
\frac{-2x + 9}{x - 1} \leq 0
\]

\[
\frac{x + 6 - 3(x - 1)}{x - 1} \leq 0
\]

\[
\frac{x + 6 - 3x + 3}{x - 1} \leq 0
\]

6) Solve using the geometric definition of absolute value (writing a sentence). Express your answer on a number line AND in interval notation:

\[
\frac{7x + 3}{7} \leq 11
\]

\[
\left| \frac{x + 3}{7} \right| \leq \frac{11}{7}
\]

7) Write the equation of a line, in point-slope form, that passes through (2,-5) and is perpendicular to the line 5x - 4y + 19 = 0.

\[
\text{slope} = -\frac{4}{5}
\]

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-5) = -\frac{4}{5}(x - 2)
\]
8) Given: \[ f(x) = \begin{cases} 
2x - 1 & \text{if } x \leq -3 \\
-4 & \text{if } -3 < x \leq 2 \\
4 - x & \text{if } 2 < x < 4
\end{cases} \]

a) Graph \( f(x) \) on the graph provided.

\[
\begin{array}{c|c}
2x-1 & 4-x \\
-3 & 2 \\
-4 & 0
\end{array}
\]

b) Using the above function, find the value of \( f(-5) - f(-3) + f(2) \).

\[
-11 - (-3) + 4 = -8
\]

9) Using the graph of \( f(x) \) shown below, write a piecewise function for \( f(x) \).

\[
f(x) = \begin{cases} 
1, & x \leq -2 \\
\frac{1}{2}x - 2, & -2 < x < 2 \\
-x + 5, & x \geq 2
\end{cases}
\]
10) Rewrite each as a piecewise function AND then sketch the graph for b, only.

\[ f(x) = \begin{cases} 
2x + 5, & x \geq -\frac{5}{2} \\
-(2x + 5), & x < -\frac{5}{2} 
\end{cases} \]

\[ \gamma = |2x + 5| \]

\[ 2x + 5 \geq 0 \\
2x \geq -5 \\
x \geq -\frac{5}{2} \]

b) \[ \gamma = \frac{6-x}{x-6} \]

\[ f(x) = \begin{cases} 
-1, & x < 6 \\
1, & x > 6 
\end{cases} \]

\[ (1, 1) \quad (5, -8) \]

11) If \( f(x) \) is a linear function and \( f(11) = 4 \) and \( f(5) = -8 \), find the value of \( f(6) \).

\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 4}{5 - 11} = \frac{-12}{-6} = 2 \]

\[ y = 2x + b \]

\[ y_1 = 2(5) + b \quad y_1 = 2(11) + b \]

\[ f(6) = 2(6) - 18 \]

12) Find the coordinates of a second point on the graph of a function \( f \) if the given point is on the graph and the function is even.

\((-3, 5) \quad (3, 5)\)

Find the coordinates of a second point on the graph of a function \( f \) if the given point is on the graph and the function is odd.

\((-1, 7) \quad (1, -7)\)
13) Determine whether the following functions are odd, even or neither. If the function is even or odd, then write a sentence using the phrase, "symmetrical to". Justify algebraically. (Use of the graph is optional.)

\[ f(x) = \frac{3x^2}{x^2 + 2} \]

\[ f(-x) = \frac{3(-x)^2}{(-x)^2 + 2} = \frac{3x^2}{x^2 + 2} \]

Even because \( f(x) = f(-x) \)  

Symmetrical to the y-axis.

14) Create three functions, \( f(x) \), \( g(x) \), and \( h(x) \), such that \( (f \circ g \circ h)(x) = \frac{6}{(x - 4)^2} \). (Note: you may not use the identity function.)

\[ h(x) = x - 4 \]

\[ g(x) = x^7 \]

\[ f(x) = \frac{6}{x} \]

15) Find the domain of the following function:

\[ f(x) = \frac{\sqrt{x - 1} + \frac{2}{\sqrt{3 - x}}}{x^2 - 1} \]

\(-1 \leq x < 3\)

\[ \frac{3 - x}{x^2 - 1} \]

\[ 3 > x \]

\[ \{ x | 1 \leq x < 3 \} = [1, 3) \]
\[
f(x) = 4x^2 - x + 3 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \frac{f(x+h) - f(x)}{h} \\
\frac{4(x+h)^2 - (x+h) + 3 - (4x^2 - x + 3)}{h} \\
\frac{4(x^2 + 2xh + h^2) - x - h + 3 - 4x^2 + x - 3}{h} \\
\frac{4x^2 + 4xh + 4h^2 - x - h}{h} - 4x^2 + x - 3 \\
\frac{1}{h} \frac{8x + 4h - 1}{h} = 8x + 4h - 1\]